Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Differential Geometry

Subject Code: 5SC02DIG1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 23/04/2018** **Time: 10:30 To 01:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. State Four vertex theorem. **(01)**

b. Is the curve $\bar{r}(t) = (t, \cos ht)$for $t \in R$ regular? **(02)**

c. Check whether the curve $\bar{r}(t) = (\frac{4}{5}cost, 1 - \sin t, -\frac{3}{5}cost)$ is planer curve or not. **(02)**

d. State Frenet – Serret formula. **(02)**

Q-2 **Attempt all questions** **(14)**

a) If $\bar{r}$ is a regular curve in R^3 then prove that the curvature $k = \frac{\|\ddot{\bar{r}} \times \dot{\bar{r}}\|}{\|\dot{\bar{r}}\|^3}$ **(06)**

b) Let$\bar{r}(t): (a, b) \rightarrow R^3$ be a regular curve. Show that $\|\bar{r}(t)\|$ is non – zero constant if and only if $\bar{r}(t) \perp \dot{\bar{r}}(t)\ \forall\ t$. **(04)**

c) Compute the arc length of the curve $\bar{r}(t) = (e^{kt}\ cost, e^{kt} sint)$ starting at the point $(1,0)$. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) Compute curvature and torsion of the curve $\bar{r}(t) = (e^t \cos t, e^t \sin t, e^t)$. **(06)**

b) Let$\bar{r}$ be a regular curve in R^3 with nowhere vanishing curvature. If $\bar{r}$ is planar then prove that the torsion of $\bar{r}$ is identically zero. **(04)**

c) Show that the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is a convex curve. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Let $\bar{r}: (a, b)\ \rightarrow R^3$ be a regular curve with nowhere vanishing curvature. Then **(06)**

prove that the torsion τ of $\bar{r}$ is $\frac{(\dot{\bar{r}}\times\ddot{\bar{r}})\cdot\dddot{\bar{r}}}{\|\dot{\bar{r}}\times\ddot{\bar{r}}\|^2}$

b) Find first fundamental form of the surface $\sigma(u,v) = (cosu\cos v, \cos u \sin v, sinu)$ **(05)**

c) In usual notation, prove that f^* is a symmetric bilinear map. **(03)**

OR

Q-3

a) State and prove Wirtinger's inequality. **(07)**

b) State and prove Iso-Perimetric inequality. **(07)**

SECTION – II

Q-4 **Define the following questions** **(07)**

a. Conformal map. **(01)**

b. Umbilical point **(01)**

c. Geodesics. **(01)**

d. Unit normal to surface. **(02)**

e. Christoffel's symbol of second kind. **(02)**

Q-5 **Attempt all questions** **(14)**

a) State and prove Euler's theorem. **(05)**

b) Let $\phi: U \to V$ be a diffeomorphism between open subsets of R^2. Let $\phi(u,v) = (f(u,v), g(u,v))$ where f and g are smooth functions. Prove that ϕ is conformal iff $(f_u = g_v \& f_v = -g_u)$or $(f_u = -g_v \& f_v = g_u)$. **(05)**

c) Find the image of Gauss map for $\sigma(u,v) = (u, v, u^2 + v^2),\ \forall\, u, v \in R$. **(04)**

OR

Q-5

a) State and prove Meusnier's theorem. **(05)**

b) Compute surface area of sphere of radius r. **(05)**

c) Let σ be a surface patch of an oriented surface with the unit normal $\bar{N}$ then prove that $\bar{N}_u\sigma_u = -L, \bar{N}_u\sigma_v = -M$ and $\bar{N}_v\sigma_v = -N$. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Calculate second fundamental form of sphere. **(06)**

b) State Gauss – Bonnet theorem. Prove that the sum of interior angles of a regular $n -$ gon on a plane is $(n-2)\pi$ **(05)**

c) Let σ be a regular surface patch. Find Γ_{11}^1. **(03)**

OR

Q-6 **Attempt all Questions**

a) Compute Gaussian curvature and mean curvature of the surface $z = f(x,y)$ where f is smooth function. **(06)**

b) Compute the principal curvature on the surface $\sigma(u,v) = (u, v, uv)$. **(05)**

c) Prove that any geodesic has a constant speed. **(03)**